the ratio of the complex amplitudes of the emergent waves from arms 2 and 3 (b_2, and b_3), be constant:

$$\frac{b_2}{b_3} = K \tag{1}$$

where K is a complex constant of finite magnitude and different from zero.

It may be noted that this condition does not actually insure that b_2, for example, will be independent of changes in the loading on arm 3. However, it does require, if a change in b_2 occurs, a simultaneous change in b_3 such that the net result is equivalent to an amplitude variation and/or phase shift in the generator to which the system is presumably insensitive.

In terms of the incident wave amplitudes (a_1, a_2, a_3), b_2 and b_3 are given by:

$$\begin{aligned} b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 \\ b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3, \end{aligned} \tag{2}$$

where the $S_{m,n}$ are the scattering coefficients of the junction.

Substituting (2) in (1) yields

$$(S_{21} - KS_{31})a_1 + (S_{22} - KS_{32})a_2 + (S_{23} - KS_{33})a_3 = 0. \tag{3}$$

In practice a_1 will not vanish since this is the emergent wave from the generator, and in the general case both a_2 and a_3 will assume arbitrary and independent values. The criterion for isolation, therefore, requires that each of the coefficients of the a's vanish.

With reference to the coefficient of a_1, if S_{21} vanishes, so must S_{31} and conversely. But this would mean that no transmission would be possible between arm 1 and either of the other two arms, and consequently represents a trivial solution. Therefore,

$$\frac{S_{21}}{S_{31}} = K.$$

Substituting this result in the coefficients of a_2 and a_3 yields

$$S_{22} - \frac{S_{21}S_{32}}{S_{31}} = 0 \tag{4}$$

$$S_{33} - \frac{S_{31}S_{23}}{S_{21}} = 0. \tag{5}$$

Eqs. (4) and (5) give the necessary and sufficient conditions that the junction satisfy (1).

In practice these conditions are approximately satisfied by a "Magic T" whose shunt or series arm has been terminated in a matched load, or by a directional coupler. In such junctions, each of the terms S_{22}, S_{23}, S_{32}, and S_{33} ideally vanish. The degree of isolation achieved when using a non-ideal hybrid junction or directional coupler can generally be improved in the following manner.

Let tuning transformers be added to each of the arms of the junction as shown in Fig. 2. The generator is removed from arm 1 and replaced by a passive load (not necessarily matched). If transformer T_1 is now adjusted so that a null[1] obtains in arm 3 with the generator connected to arm 2, it can be shown that the reflection coefficient "looking into" arm 2 is just the left hand member of (4). Transformer T_2 is now adjusted so that this reflection coefficient vanishes, and (4) is satisfied. It may be intuitively recognized that this adjustment is independent of the subsequent adjustment of transformer T_3 since if a null exists in arm 3, the reflection coefficient observed at arm 2 will evidently be independent of the manner of terminating arm 3. The interchange of arms 2 and 3 in the above procedure satisfies (5). (If, as is frequently the case, the load terminating one of the arms, arm 3 for example, is constant or not subject to variations in impedance, then it can be shown that this second step or the adjustment of the transformer in the arm terminated by the fixed load is not required.) Once the proper adjustment of T_2 and T_3 has been realized, the adjustment of T_1 is no longer important since the conditions expressed by (4) and (5) are invariant to the adjustment of T_1. If, as an additional condition, T_1 is terminated by a load equal to the generator impedance during the tuning procedure, then b_2 and b_3 individually, as well as their ratio, will remain constant. In many applications this is of at least nominal interest.

The extension of this procedure to three or more channels is straightforward. In the four arm junction of Fig. 3, for example, let the arm which will ultimately be connected to the generator (arm 1) be connected to an arbitrary load as previously. If now by means of tuner T_1 and other "internal" tuning adjustments (which will in general be required), it is possible to achieve the condition where no coupling exists between the remaining arms (no output is observed at arms 3 and 4 with arm 2 connected to the generator, etc.) and if tuners T_2, T_3, T_4, have been adjusted so that the reflection coefficients at the respective arms vanish, then it can be shown that the desired condition obtains. The considerations involving tuner T_1, as outlined above, also continue to apply.

One possible method of obtaining such a junction is shown in Fig. 4, where tuner T_a provides the "internal" adjustment as required above.

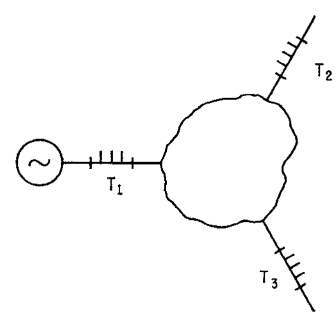

Fig. 2—Three arm junction with tuners.

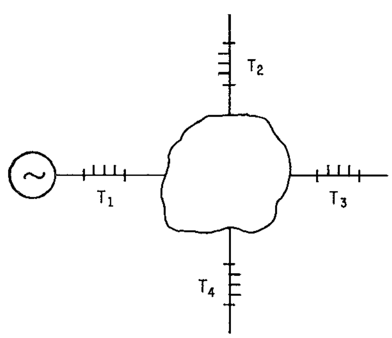

Fig. 3—Four arm junction and tuners.

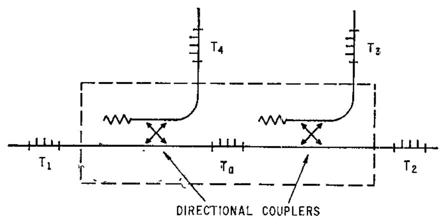

Fig. 4—Four arm junction with internal tuning.

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[1] If transformer T_1 is assumed to be dissipationless, this adjustment may be realized provided that the junction satisfies the condition

$$\left| S_{11} - \frac{S_{12}S_{31}}{S_{32}} \right| > 1.$$

A Microwave Impedance Meter Capable of High Accuracy*

Recent applications of directional couplers with auxiliary tuners for accurate VSWR and phase shift measurements have made possible a microwave impedance meter capable of high accuracy. It is similar to some bridges in that one obtains an initial detector null and a final null, before and after connecting the unknown. Both the magnitude and phase angle of the reflection coefficient of the unknown are determined in this operation, and these can be made direct reading if desired. The principle of operation is as follows.

The use of directional couplers with auxiliary tuners permits adjustment for the conditions $S_{31} = \Gamma_{2i} = 0$, whereupon

$$b_3 = C\Gamma_L. \tag{1}$$

The symbols have the following meanings, which become clearer upon reference to Fig. 1.

S_{31} = transmission (scattering) coefficient for waves going from arm 1 to arm 3,

Γ_{2i} = reflection coefficient which would be measured "looking into" arm 2 if the generator were replaced by a passive impedance having the same impedance as the generator,

b_3 = amplitude of wave emerging from arm 3,

C = a constant which depends upon the parameters of the adjusted directional coupler-tuner combination,

* Received by the PGMTT, April 1, 1960.

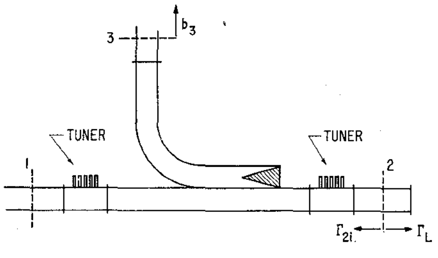

Fig. 1.

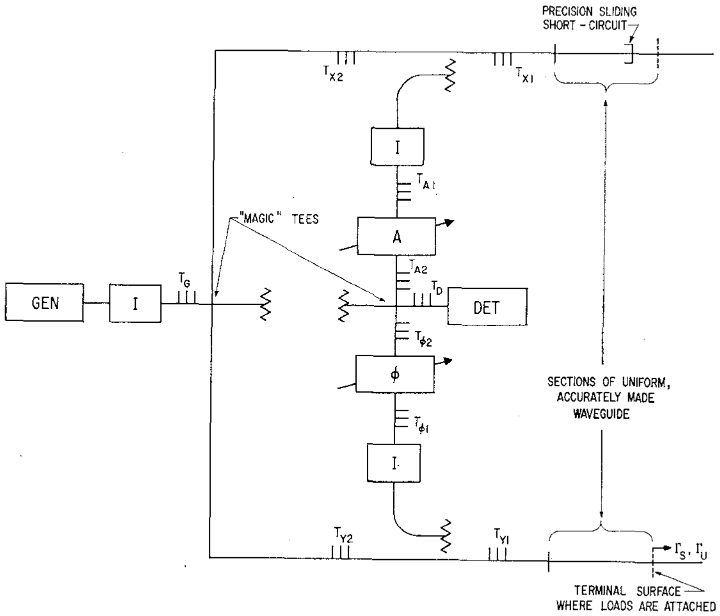

Fig. 2.

Γ_L = reflection coefficient of load terminating arm 2.

The tuning of a junction to obtain the performance indicated by (1) is the basis for a reflectometer[1] to determine $|\Gamma_L|$ and for a phase shift standard[2] which makes use of the fact that the phase of b_3 tracks the phase of Γ_L.

A microwave impedance meter can be constructed by a combination of the two ideas. A way in which this can be assembled is shown in Fig. 2.

After proper adjustment of the tuners, one adjusts the calibrated attenuator and sliding short circuit to obtain detector nulls with a standard known (Γ_s) termination and the unknown (Γ_u) alternately connected to the place indicated in Fig. 2. The variable phase shifter shown in the lower arm is not essential but may be used for convenience in zero-setting the short circuit or in making rapid, direct-reading phase measurements.

One calculates the magnitude of Γ_u from the increase in attenuation ΔA required to restore the null when Γ_s is replaced by Γ_u,

$$\Delta A = 20 \log_{10} \left| \frac{\Gamma_s}{\Gamma_u} \right| . \tag{2}$$

The phase ψ_u of Γ_u, the reflection coefficient of the unknown, equals the phase shift produced by the sliding short circuit plus the phase ψ_s of Γ_s,

$$\psi_u = 2\beta l + \psi_s. \tag{3}$$

In this equation, it is apparent that l is the displacement toward T_{X1} of the short-circuit and β the phase constant of the waveguide containing the sliding short circuit. If the phase shifter ϕ is varied instead of the short circuit, one substitutes its phase change for $2\beta l$ in (3).

The following assumptions implicit in the above procedure are noted:

1) Adjustment of the attenuator does not produce any change in phase. (Attenuators are commercially available which come close to meeting this requirement, and in principle, one could measure their phase shift and deduct it.)
2) Adjustments of the attenuator and phase shifter in the side arms of the couplers do not disturb the adjustments of the tuners to make $\Gamma_{2i} = 0$. (The isolators labeled I in the diagram are suggested to reduce trouble from this possibility.)
3) The complex ratio of the amplitudes of the waves entering the two channels is not affected by changes of loading occurring during a measurement. (The decoupling offered by the hybrid tees or 3-db directional couplers may be augmented by the auxiliary tuners T_{X2} and T_{Y2} shown in Fig. 2; the isolation can be made arbitrarily high at any particular operating frequency.)

Suitable standards of reflection are a) quarter-wavelength short circuits,[3] in which case $\Gamma_s = 1$, (2) yields the return loss $\Delta A = 20 \log_{10} 1/|\Gamma_u|$, and (3) yields the phase $\psi_u = 2\beta l$ directly, and b) half-round inductive obstacle impedance standards,[4] *i.e.*, obstacles having accurately calculable magnitude and phase of reflection, in which Γ_s may be chosen close to Γ_u, thereby reducing the error in the measurement.

The adjustments of the tuners to obtain the best accuracy with the components and instrumentation available are adaptations of known techniques[1] with the exception of steps 1) and 5) which were originated by G. F. Engen as a modification of a more general tuning procedure.[5] The steps in the tuning procedure are as follows:

1) With generator connected to the uniform waveguide section which normally contains the sliding short circuit, and a detector connected to the other uniform waveguide section, adjust T_G and T_D separately for a detector null, blocking in turn the paths from T_D, then T_G, to the detector. (Switches may be permanently installed in the waveguide system for this purpose, if desired.)
2) Temporarily remove the attenuator A and adjust T_{A1} and T_{A2} for no reflection. (This condition may be recognized with an auxiliary reflectometer.)
3) Temporarily remove the phase shifter ϕ, and adjust $T_{\phi 1}$ and $T_{\phi 2}$ for no reflection.
4) Adjust T_{X1} and T_{Y1} for infinite directivity of the directional couplers and associated tuners. Two ways will be described for doing this. a) Introduce a nonreflecting termination into the waveguide section connected to the tuner and adjust the tuner until a null is obtained in the sidearm output. b) Introduce a weakly reflecting (VSWR < 1.01) termination into the waveguide section connected to the tuner, and while sliding it back and forth in the waveguide, adjust the tuner until there are no cyclical variations in the sidearm output. This procedure will not in itself give infinite directivity but in practice will usually result in a very high value.[1]
5) With the generator connected as shown in Fig. 2, a) adjust T_{X2} so that sliding a short circuit in the uniform waveguide section in channel X produces no cyclical variation in the detector output, and b) adjust T_{Y2} so that sliding a short circuit in the uniform waveguide section in channel Y produces no cyclical variation in the detector output.

When the above adjustments have been completed, the two channels are well isolated, the two directional coupler and tuner combinations have approximately infinite directivity, and a nonreflecting equivalent generator at their outputs in the uniform waveguide sections; and the calibrated attenuator and phase shifter are connected to

[1] G. F. Engen and R. W. Beatty, "Microwave reflectometer techniques," IRE Trans. on Microwave Theory and Techniques, vol. 7, pp. 351–355; July, 1959.

[2] M. Magid, "Precision microwave phase shift measurements," IRE Trans. on Instrumentation, vol. 7, pp. 321–331; December, 1958.

[3] R. W. Beatty and D. M. Kerns, "Recently developed microwave impedance standards and methods of measurement," IRE Trans. on Instrumentation, vol. 7, pp. 319–231; December, 1958.

[4] D. M. Kerns, "Half-round inductive obstacles in rectangular waveguide," to be published in *J. Res. NBS*, Section B, Mathematics and Mathematical Phys.

[5] G. F. Engen, "A method of improving isolation in multi-channel waveguide systems," this issue, p. 460.

nonreflecting waveguides, preventing mismatch errors.

Standards of phase shift and impedance may be made to high precision by careful machining techniques. With refined instrumentation, the system should prove capable of impedance measurements of the highest accuracy and therefore useful in calibration work.

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On the Noise Temperature of Coupling Networks*

When a passive coupling network, such as a waveguide, transmission line, matching filter, etc., is used to connect a source to a receiver, it is apparent that it will contribute noise to the output because of its lossiness. If the noise temperature of the source is T_s and the temperature of the coupling network is T_n, then the noise temperature, T_o, at the output (under matched conditions) is given by[1]

$$T_o = \frac{T_s}{L} + T_n\left(1 - \frac{1}{L}\right), \qquad (1)$$

where L is the coupling-network power loss ratio. This relationship was derived by constructing a transmission line analog to the coupling network and treating the source and loss noises as propagating signals. An alternative derivation based on a more physical representation is presented in this note.

Consider the coupling network as a generalized two-port with matched input and output. Its noise power output, P, can be written

$$P = \frac{kT_sB}{L} + kT_nBf, \qquad (2)$$

where the first term is simply the attenuated source noise power and the other is some fraction, f, of the noise power available from the coupling network. Since (2) is true for all values of the parameters, it is true, in particular, when the coupling network is at the same temperature as the source, yielding

$$P_{T_n=T_s} = kT_sB\left(\frac{1}{L} + f\right). \qquad (3)$$

However, the noise contributions to the output from the source and from the coupling network become indistinguishable when both are at the same temperature. That is, the output from the coupling network then looks exactly like that from the source itself, so

$$P_{T_n=T_s} = kT_sB, \qquad (4)$$

and combining (3) and (4) yields

$$f = 1 - \frac{1}{L}. \qquad (5)$$

Writing the general noise power output, P, as kT_0B then gives

$$T_0 = \frac{T_s}{L} + T_n\left(1 - \frac{1}{L}\right), \qquad (6)$$

Q.E.D.

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* Received by the PGMTT, April 1, 1960.

[1] P. D. Strum, "A note on noise temperature," IRE TRANS. ON MIROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 145–151; July, 1956.

A Logarithmic Transmission Line Chart*

In his article above,[1] Hudson raises the question: "What length of line of what impedance will match a given impedance?" He states, "Conventional charts do not answer this question explicitly."

This problem can be solved on the "conventional" Smith Chart (Fig. 1) explicitly without trial-and-error, by the following method. If A and B are two quite general impedances, the matched condition requires that A be transformed into B^*, the complex conjugate of B.

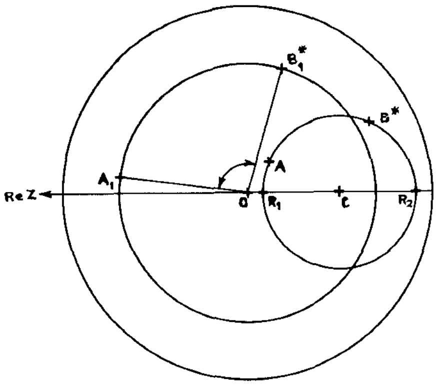

Fig. 1—Smith Chart.

1) Plot A and B^* on the Smith Chart and draw the circle through these two points which has its center (C) on the real axis. If this circle lies fully within the Smith Chart, the question has a solution, otherwise not.
2) Read off the values at the intersections of the real axis and the circle (R_1, R_2), and determine their geometric mean $\sqrt{R_1 \times R_1}$, which will be the characteristic impedance of the matching line.

To find the length of this line,

3) On the Smith Chart normalized to $\sqrt{R_1 \times R_2}$, represent A and B^* by A_1 and B_1^*.
4) The electrical length of the matching line will be given by half of the angle $A_1OB_1^*$.

This method is based upon the following two properties of loss-free transmission lines:

1) The locus of the impedance along a line is always a circle on the Smith Chart (having its centre on the real axis) irrespective of the value of the normalizing resistance.
2) The characteristic impedance of the line is given by $Z_0 = \sqrt{(\text{Re } Z)_{\max} \times (\text{Re } Z)_{\min}}$, where Z is the impedance along the line.

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* Received by the PGMTT, April 1, 1960.

[1] A. C. Hudson, IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 277–281; April, 1959.

Velocity Sorting Detection in Backward Wave Autodyne Reception*

An electronically tunable microwave receiver which uses an oscillating backward wave amplifier driving a crystal detector has been described previously.[1] This receiver has the advantages of large dynamic range and good rejection of unwanted signals, but has the disadvantage that its frequency response can be no better than that of its crystal detector. Since a variation in sensitivity of greater than 3 db over the tuning range of 8 to 12 kmc would seriously lower its usefulness as a spectrum display device, the restrictions on the crystal detector performance are quite severe.

In the paper describing the operation of the device, the author made the suggestion that it might be possible to detect the video output by means of a suitable collector. This letter describes the results of an experimental velocity sorting detector used with the backward wave autodyne receiver.

The first tube used was a Varian VAD-161-2. The collector in this tube was not designed for depressed operation, and, as a result, when the collector voltage was lowered to within a few volts of the cathode potential a virtual cathode was formed near the collector.

A three-dimensional plot of collector current vs collector voltage and beam current is presented in Fig. 1. The current is a multivalued function of collector potential which resulted in the production of oscillations when a load resistance was connected to the collector. Because the oscillations occurred at the setting of collector voltage

* Received by the PGMTT, December 28, 1959; revised manuscript received, April 4, 1960.

[1] J. K. Pulfer, "Application of a backward wave amplifier to microwave autodyne reception," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 356–359; July, 1959.